# Solubility Equilibrium & Solubility Product

## Calculating Ksp

1. $AgBr_{(s)} \rightleftharpoons Ag^+_{(aq)} + Br^-_{(aq)}$ $\quad x = 8.8 \times 10^{-7}$ mol/L

[E] $\quad 8.8 \times 10^{-7} \quad 8.8 \times 10^{-7}$

$K_{sp} = [Ag^+][Br^-] = (8.8 \times 10^{-7})^2 = 7.7 \times 10^{-13}$

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2. $Fe(OH)_{2(s)} \rightleftharpoons Fe^{2+}_{(aq)} + 2OH^-_{(aq)}$ $\quad x = 1.4 \times 10^{-3}$ g/L

[E] $\quad 1.56 \times 10^{-5} \quad 2(1.56 \times 10^{-5})$ $\quad = 1.56 \times 10^{-5}$ mol/L

$K_{sp} = [Fe^{2+}][OH^-]^2 = (1.56 \times 10^{-5})(3.11 \times 10^{-5})^2 = 1.5 \times 10^{-14}$

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3. $CuBr_{2(s)} \rightleftharpoons Cu^{2+}_{(aq)} + 2Br^-_{(aq)}$ $\quad x = 2.0 \times 10^{-4}$ mol/L

[E] $\quad 2.0 \times 10^{-4} \quad 4.0 \times 10^{-4}$

$K_{sp} = [Cu^{2+}][Br^-]^2 = (2.0 \times 10^{-4})(4.0 \times 10^{-4})^2 = 3.2 \times 10^{-11}$

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4. $SrSO_{4(s)} \rightleftharpoons Sr^{2+}_{(aq)} + SO_4^{2-}_{(aq)}$ $\quad x = 5.83 \times 10^{-4}$ mol/L

[E] $\quad 5.83 \times 10^{-4} \quad 5.83 \times 10^{-4}$

$K_{sp} = [Sr^{2+}][SO_4^{2-}] = (5.83 \times 10^{-4})^2 = 3.4 \times 10^{-7}$

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5. $Mg(OH)_{2(s)} \rightleftharpoons Mg^{2+}_{(aq)} + 2OH^-_{(aq)}$ $\quad x = 1.5 \times 10^{-4}$ mol/150mL

[E] $\quad 1.5 \times 10^{-3} \quad 3.0 \times 10^{-3}$ $\quad = 1.5 \times 10^{-3}$ mol/L

$K_{sp} = [Mg^{2+}][OH^-]^2 = (1.5 \times 10^{-3})(3.0 \times 10^{-3})^2 = 1.4 \times 10^{-8}$

6. $Ag_2S_{(s)} \rightleftharpoons 2Ag^+_{(aq)} + S^{2-}_{(aq)}$ $\quad x = 1.3 \times 10^{-3}$ mol / 50 mL

[E] $\quad 5.2 \times 10^{-2} \quad 2.6 \times 10^{-2}$ $\quad = 2.6 \times 10^{-2}$ mol/L

$K_{sp} = [Ag^+]^2[S^{2-}] = (5.2 \times 10^{-2})^2(2.6 \times 10^{-2}) =$ **$7.0 \times 10^{-5}$**

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7. $BaF_{2(s)} \rightleftharpoons Ba^{2+}_{(aq)} + 2F^-_{(aq)}$ $\quad x = 2.2 \times 10^{-2}$ mol/L

[E] $\quad 2.2 \times 10^{-2} \quad 4.4 \times 10^{-2}$

$K_{sp} = [Ba^{2+}][F^-]^2 = (2.2 \times 10^{-2})(4.4 \times 10^{-2})^2 =$ **$4.3 \times 10^{-5}$**

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8. $Li_2CO_{3(s)} \rightleftharpoons 2Li^+_{(aq)} + CO_3^{2-}_{(aq)}$ $\quad x = 1.5$ g / 100 mL

[E] $\quad 0.40 \quad 0.20$ $\quad = 15$ g/L

$= 0.20$ mol/L

$K_{sp} = [Li^+]^2[CO_3^{2-}] = (0.40)^2(0.20)$

$=$ **0.032**

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## Calculating solubility from $K_{sp}$

1. $CuCO_{3(s)} \rightleftharpoons Ca^{2+}_{(aq)} + CO_3^{2-}_{(aq)}$ $\quad K_{sp} = 4.8 \times 10^{-5}$

| | | |
|---|---|---|
| I | 0 | 0 |
| C | +x | +x |
| E | x | x |

4.8 $K_{sp} = [Ca^{2+}][CO_3^{2-}]$

$4.8 \times 10^{-5} = x^2$

$x = 6.9 \times 10^{-3}$

∴ solubility is **$6.9 \times 10^{-3}$ mol/L**

2. $Ca(OH)_{2(s)} \rightleftharpoons Ca^{2+}_{(aq)} + 2OH^-_{(aq)}$ $K_{sp} = 4.0 \times 10^{-6}$

| | $Ca(OH)_2$ | $Ca^{2+}$ | $OH^-$ |
|---|---|---|---|
| I | 1 | 0 | 0 |
| C | | $+x$ | $+2x$ |
| E | | $x$ | $2x$ |

$K_{sp} = [Ca^{2+}][OH^-]^2$

$4.0 \times 10^{-6} = (x)(2x)^2$ ∴ solubility is $1.0 \times 10^{-2}$ mol/L

$4.0 \times 10^{-6} = 4x^3$ OR 0.74 g/L

$x = 1.0 \times 10^{-2}$

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3. $MgF_{2(s)} \rightleftharpoons Mg^{2+}_{(aq)} + 2F^-_{(aq)}$ $K_{sp} = 6.4 \times 10^{-9}$

| | $MgF_2$ | $Mg^{2+}$ | $F^-$ |
|---|---|---|---|
| I | 1 | 0 | 0 |
| C | | $+x$ | $+2x$ |
| E | | $x$ | $2x$ |

$K_{sp} = [Mg^{2+}][F^-]^2$ ∴ solubility is $1.2 \times 10^{-3}$ mol/L

$6.4 \times 10^{-9} = (x)(2x)^2 = 4x^3$ OR 0.075 g/L

$x = 1.2 \times 10^{-3}$

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4. [E] $CuS_{(s)} \rightleftharpoons Cu^{2+}_{x\,(aq)} + S^{2-}_{x\,(aq)}$ $K_{sp} = 4.0 \times 10^{-38}$

a) $4.0 \times 10^{-38} = x^2$ $2.0 \times 10^{-19}$ moles will dissolve in 1.0 L

$x = 2.0 \times 10^{-19}$

b) $m = (2.0 \times 10^{-19})(95.61)$ $1.9 \times 10^{-17}$ g will dissolve in 1.0 L

c) $[S^{2-}] = 2.0 \times 10^{-19}$ mol/L

5. $Ca(OH)_{2(s)} \rightleftharpoons Ca^{2+}_{(aq)} + 2OH^-_{(aq)}$ $K_{sp} = 4 \times 10^{-6}$

E: $x$ $2x$

a) $4 \times 10^{-6} = (x)(2x)^2 = 4x^3$

$x = 1 \times 10^{-2}$

$1 \times 10^{-2}$ mol/L $\times$ 74.08 g/mol = 0.74 g/L

b) $[OH^-] = 2(1 \times 10^{-2}) =$ $2 \times 10^{-2}$ mol/L

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6. $Ag_2CrO_{4(s)} \rightleftharpoons 2Ag^+_{(aq)} + CrO_4^{2-}_{(aq)}$ $K_{sp} = 5.00 \times 10^{-13}$

E: $2x$ $x$

$5.00 \times 10^{-13} = (2x)^2 x = 4x^3$

$x = 5.00 \times 10^{-5}$

$\therefore [Ag^+] =$ $1.00 \times 10^{-4}$ mol/L

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7. $Ag_2CO_{3(s)} \rightleftharpoons 2Ag^+_{(aq)} + CO_3^{2-}_{(aq)}$ $K_{sp} = 8.20 \times 10^{-12}$

E: $2x$ $x$

$8.20 \times 10^{-12} = (2x)^2(x) = 4x^3$

$x = 1.27 \times 10^{-4}$ mol/L

$[Ag^+] = 2(1.27 \times 10^{-4}) = 2.54 \times 10^{-4}$

fix → ~~$1.27 \times 10^{-4} \times 275.81 = 3.50 \times 10^{-2}$ g/L~~

$(2.54 \times 10^{-4})(107.9) = 2.74 \times 10^{-2}$ g/L

# WAPF

1. $CaSO_4(s) \rightleftharpoons Ca^{2+}_{(aq)} + SO_4^{2-}_{(aq)}$

[I] $\quad 0.0050 \qquad 0.040$

$$[CaCl_2] = \frac{0.010 \times 20}{40} = 0.0050\,M = [Ca^{2+}]$$

$$[Na_2SO_4] = \frac{0.080 \times 20}{40} = 0.040\,M = [SO_4^{2-}]$$

$Q = (0.0050)(0.040) =$ **$2.0 \times 10^{-4}$** $\quad Q > K_{sp}$ ∴ **yes**

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2. $MgCO_3(s) \rightleftharpoons Mg^{2+}_{(aq)} + CO_3^{2-}_{(aq)} \quad K_{sp} = 2.6 \times 10^{-5}$

[I] $\quad 3.2 \times 10^{-3} \qquad 6.0 \times 10^{-3}$

$$[Mg(NO_3)_2] = \frac{8.0 \times 10^{-3} \times 40.0}{100.0} = 3.2 \times 10^{-3}\,M = [Mg^{2+}]$$

$$[K_2CO_3] = \frac{1.0 \times 10^{-2} \times 60.0}{100.0} = 6.0 \times 10^{-3}\,M = [CO_3^{2-}]$$

$Q = (3.2 \times 10^{-3})(6.0 \times 10^{-3}) =$ **$1.9 \times 10^{-5}$** $\quad Q < K_{sp}$ ∴ **NO**

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3. $Ag_2CrO_4(s) \rightleftharpoons 2Ag^+_{(aq)} + CrO_4^{2-}_{(aq)} \quad K_{sp} = 9.0 \times 10^{-12}$

$\qquad\qquad 1.0 \times 10^{-3} \qquad 1.5 \times 10^{-4}$

$$[AgNO_3] = \frac{4 \times 10^{-3} \times 25}{100} = 1 \times 10^{-3}\,M = [Ag^+]$$

$$[Na_2CrO_4] = \frac{2.0 \times 10^{-4} \times 75}{100} = 1.5 \times 10^{-4}\,M = [CrO_4^{2-}]$$

$Q = (1.0 \times 10^{-3})^2 (1.5 \times 10^{-4}) =$ **$1.5 \times 10^{-10}$**

$Q > K_{sp}$ ∴ **yes**

4. $PbSO_{4(s)} \rightleftharpoons Pb^{2+}_{(aq)} + SO_4^{2-}_{(aq)}$ $K_{sp} = 6.3 \times 10^{-7}$

[I] $5.01 \times 10^{-4}$ 0.01

$n_{Pb(NO_3)_2} = \frac{0.166\,g}{331.22} = 5.01 \times 10^{-4}$ mol in 1.0 L

$[Pb^{2+}] = 5.01 \times 10^{-4}$ mol/L

$[Na_2SO_4] = 0.01\,M = [SO_4^{2-}]$

$Q = (5.01 \times 10^{-4})(0.01) = 5.01 \times 10^{-6}$

$Q > K_{sp}$ ∴ yes.

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5. $AgCl_{(s)} \rightleftharpoons Ag^+_{(aq)} + Cl^-_{(aq)}$ $K_{sp} = 1.8 \times 10^{-10}$

$1.0 \times 10^{-14}$ $5 \times 10^{-5}$

$[NaCl] = \frac{1 \times 10^{-4} \times 25}{50} = 5 \times 10^{-5}\,M = [Cl^-]$

$[AgNO_3] = \frac{2 \times 10^{-14} \times 25}{50} = 1 \times 10^{-14} = [Ag^+]$

$Q = (1.0 \times 10^{-14})(5 \times 10^{-5}) = 5 \times 10^{-19}$

$Q < K_{sp}$ ∴ NO.

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6. $AgBr_{(s)} \rightleftharpoons Ag^+_{(aq)} + Br^-_{(aq)}$ $K_{sp} = 5.0 \times 10^{-13}$

$7.5 \times 10^{-3}$ $1.0 \times 10^{-6}$

$[NaBr] = \frac{4.0 \times 10^{-6} \times 25}{100} = 1.0 \times 10^{-6} M = [Br^-]$

$[AgNO_3] = \frac{1.0 \times 10^{-2} \times 75}{100} = 7.5 \times 10^{-3} M = [Ag^+]$

$Q = (7.5 \times 10^{-3})(1.0 \times 10^{-6}) = 7.5 \times 10^{-9}$

$Q > K_{sp}$ ∴ yes.

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7. $CuBr_{(s)} \rightleftharpoons Cu^+_{(aq)} + Br^-_{(aq)}$ $K_{sp} = 5.9 \times 10^{-9}$

$3.0 \times 10^{-11}$ $2.8 \times 10^{-6}$

$[NaBr] = \frac{4.0 \times 10^{-6} \times 70}{100} = 2.8 \times 10^{-6} M = [Br^-]$

$[CuNO_3] = \frac{1.0 \times 10^{-10} \times 30}{100} = 3.0 \times 10^{-11} = [Cu^+]$

$Q = (3.0 \times 10^{-11})(2.8 \times 10^{-6}) = 8.4 \times 10^{-17}$

$Q < K_{sp}$ ∴ NO.

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Common Ion Effect

1. $AgCl_{(s)} \rightleftharpoons Ag^+_{(aq)} + Cl^-_{(aq)}$ $K_{sp} = 2.0 \times 10^{-10}$

$[NaCl] = 0.10 M = [Cl^-]$

| | | $Ag^+$ | $Cl^-$ |
|---|---|---|---|
| I | | 0 | 0.10 |
| C | | +x | +x |
| E | | x | 0.10 + x |

Note: in water $x = 1.4 \times 10^{-5}$ ∴ x is insignificant

$2.0 \times 10^{-10} = x(0.10)$

$x = 2.0 \times 10^{-9}$

∴ the solubility in NaCl is $2.0 \times 10^{-9}$ mol/L

2. $SrSO_{4(s)} \rightleftharpoons Sr^{2+}_{(aq)} + SO_4^{2-}_{(aq)}$

| | | | |
|---|---|---|---|
| I | | 0 | 0.20 |
| C | | $+x$ | $+x$ |
| E | | $x$ | $0.20 + x$ |

Soly in water = $8.5 \times 10^{-5}$

in CI

$x << 8.5 \times 10^{-5}$

$\therefore$ insignificant

$7.2 \times 10^{-9} = x(0.20)$

$x = 3.6 \times 10^{-8}$

$\therefore$ soly in $Na_2SO_4$ is $3.6 \times 10^{-8}$ mol/L

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3. $CaF_{2(s)} \rightleftharpoons Ca^{2+}_{(aq)} + 2F^-_{(aq)}$ $K_{sp} = 4.0 \times 10^{-11}$

| | | | |
|---|---|---|---|
| I | | 0 | 0.025 |
| C | | $+x$ | $+2x$ |
| E | | $x$ | $0.025 + 2x$ |

Soly in water:

$4.0 \times 10^{-11} = 4x^3$

$x = 3 \times 10^{-4}$

$\therefore x << 3 \times 10^{-4}$

$\therefore$ insign.

$4.0 \times 10^{-11} = (x)(0.025)^2$

$x = 6.4 \times 10^{-8}$

$\therefore$ soly of $CaF_2$ in NaF is $6.4 \times 10^{-8}$ M/L

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4. $Ag_3PO_{4(s)} \rightleftharpoons 3Ag^+_{(aq)} + PO_4^{3-}_{(aq)}$ $K_{sp} = 1.8 \times 10^{-18}$

| | | |
|---|---|---|
| E | $3x$ | $1.0 \times 10^{-5}$ |

$1.8 \times 10^{-18} = (x)^3(1.0 \times 10^{-5})$

$x = 5.6 \times 10^{-5} = [Ag^+]$ at equil - any value greater than this will cause a precipitate.

$\therefore [AgNO_3] > 5.6 \times 10^{-5}$ M/L